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Clustering of chiral particles in flows with broken parity invariance

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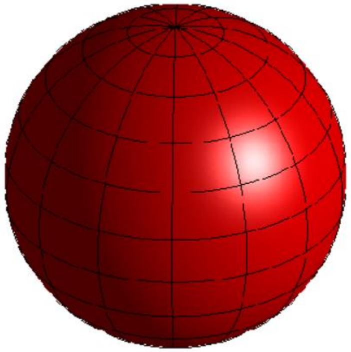
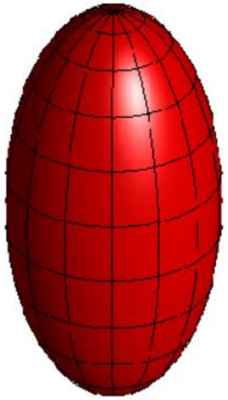
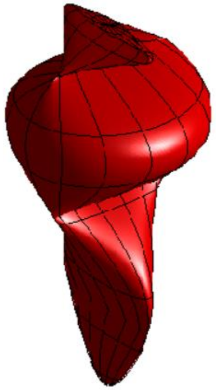
Particle symmetries

Reflection invariance \ Rotation invariance	✓	✗
✓		
✗	**‘Isotropic helicoid’** (this talk)	

Example of an isotropic helicoid

Recipe from Lord Kelvin:

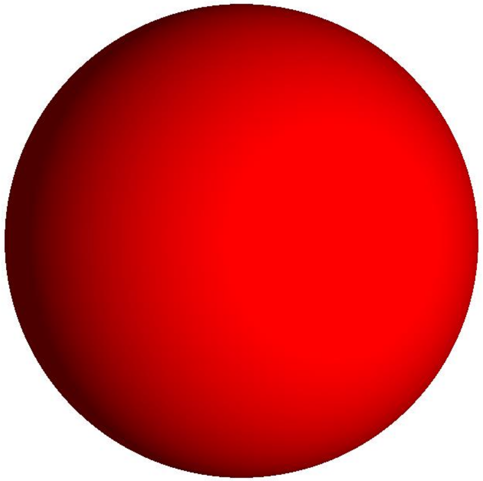
"An isotropic helicoid can be made by attaching projecting vanes to the surface of a globe in proper positions; for instance cutting at 45° each, at the middles of the twelve quadrants of any three great circles dividing the globe into eight quadrantal triangles."

Kelvin, Phil. Mag. **42** (1871)

Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

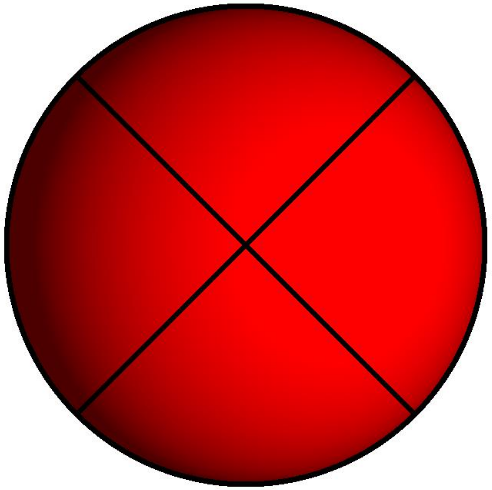
Start with a sphere



Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

✓ Start with a sphere
Draw 3 great circles

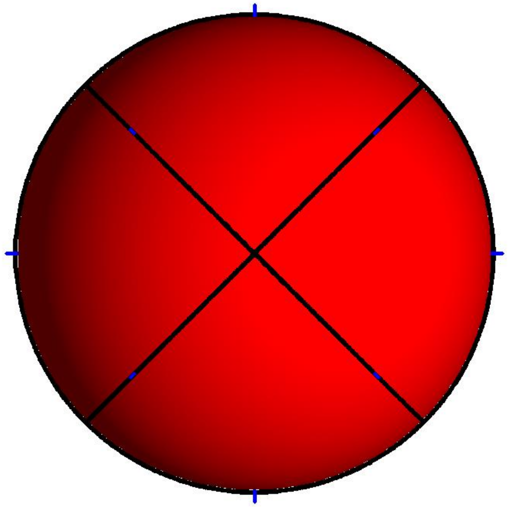


Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

- ✓ Start with a sphere
- ✓ Draw 3 great circles

Identify 12 vane positions at midpoints of quarter-arcs

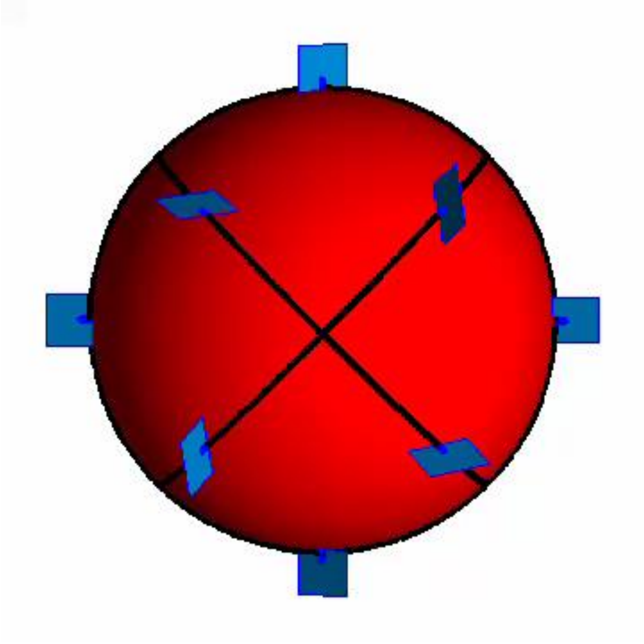


Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

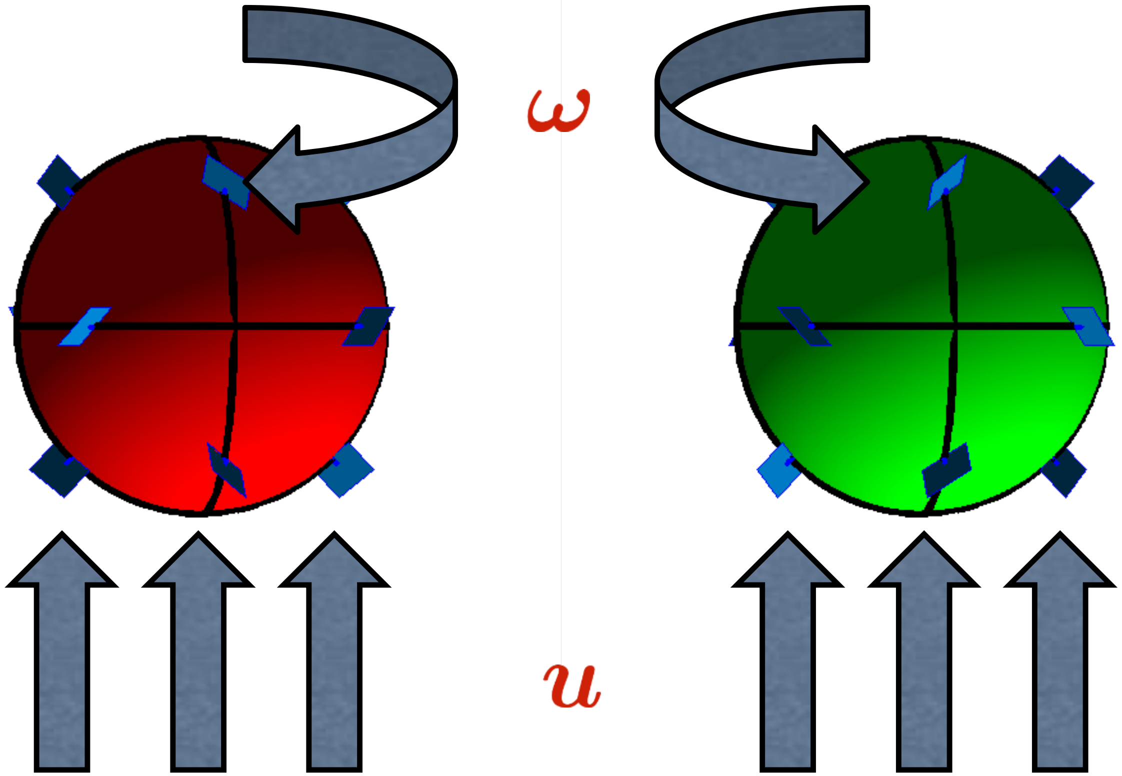
- ✓ Start with a sphere
- ✓ Draw 3 great circles
- ✓ Identify 12 vane positions at midpoints of quarter-arcs

Put a vane on each vane position (45° to arc line)



Chirality

In a constant flow u, the isotropic helicoid starts spinning around the flow direction with angular velocity ω.
The spinning direction depends on the chirality of the vanes.



Left handed $C_0 < 0$ *Right handed* $C_0 > 0$

Motion of an ‘isotropic helicoid’

Equations for velocity $\boldsymbol{v}$ and angular velocity $\boldsymbol{\omega}$ for small isotropic helicoid:

Happel & Brenner, Low Reynolds number hydrodynamics (1963)

$$\dot{\boldsymbol{v}} = \frac{1}{\tau_{\mathrm{p}}}\left[\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{v} + \frac{2a}{9}C_0(\boldsymbol{\Omega}(\boldsymbol{r},t) - \boldsymbol{\omega})\right]$$

$$\dot{\boldsymbol{\omega}} = \frac{1}{\tau_{\mathrm{p}}}\left[\frac{10}{3}(\boldsymbol{\Omega}(\boldsymbol{r},t) - \boldsymbol{\omega}) + \frac{5}{9a}C_0(\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{v})\right]$$

Stokes’ law translation – rotation coupling (scalar)

$a = \sqrt{5I_0/(2m)}$ Particle ‘size’ (defined by mass m and moment of inertia I_0)

C_0 Helicoidality

Ratio of rotational and translational inertia fixed to that of sphere

Equations break spatial reflection symmetry ($\boldsymbol{\omega}$ pseudovector)

Dimensionless parameters

Stokes number $\mathrm{St} \equiv \frac{\tau_\mathrm{p}}{\tau_\eta}$ Size $\overline{a} \equiv \frac{a}{\eta}$ Helicoidality C_0

with τ_η and η smallest time- and length scales of flow.

Dynamics may grow indefinitely unless $-\sqrt{27} < C_0 < \sqrt{27}$.

Simulations and theory is done using a random single-scale flow characterised by the Kubo number

$$\mathrm{Ku} \equiv \frac{u_0 \tau_\eta}{\eta}$$

with u_0 typical speed of flow.

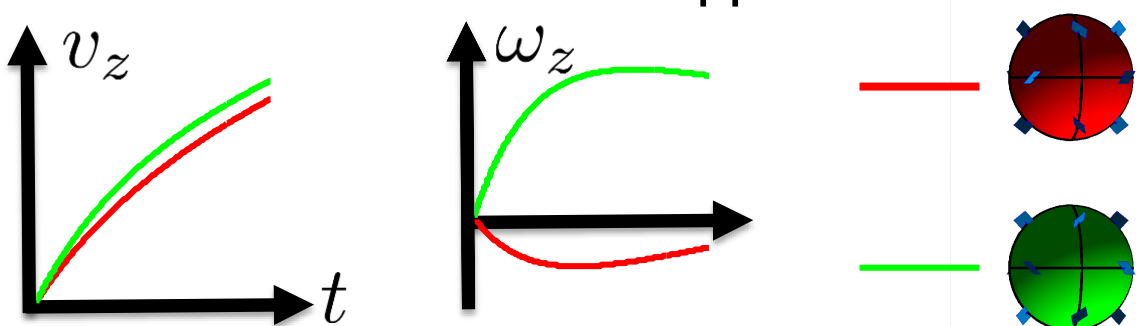
Chiral dynamics

$$\dot{\boldsymbol{v}} = \frac{1}{\tau_{\mathrm{p}}}\left[\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{v} + \frac{2a}{9}C_0(\boldsymbol{\Omega}(\boldsymbol{r},t) - \boldsymbol{\omega})\right]$$

$$\dot{\boldsymbol{\omega}} = \frac{1}{\tau_{\mathrm{p}}}\left[\frac{10}{3}(\boldsymbol{\Omega}(\boldsymbol{r},t) - \boldsymbol{\omega}) + \frac{5}{9a}C_0(\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{v})\right]$$

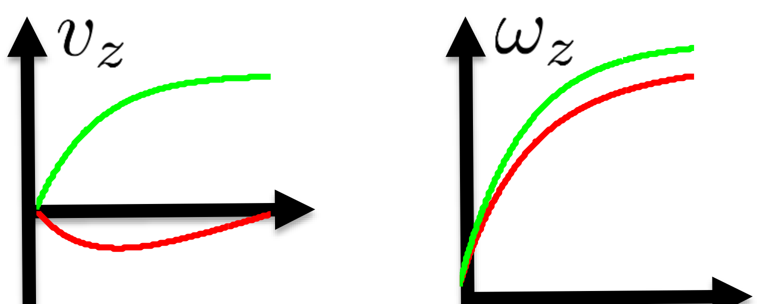
When $\boldsymbol{\Omega} = 0$ the translational dynamics is independent of $\mathrm{sign}(C_0)$.
$\Rightarrow$ In straining regions of the flow ($\Omega \approx 0$) both chiralities have approximately same center-of mass motion, but rotate in opposite directions.

$|\Omega_z| \ll |u_z|$



When $\boldsymbol{u} = 0$ the rotational dynamics is independent of $\mathrm{sign}(C_0)$.
$\Rightarrow$ In elliptic regions of the flow ($\boldsymbol{u} \approx 0$) both chiralities have approximately same rotation, but move in opposite directions.

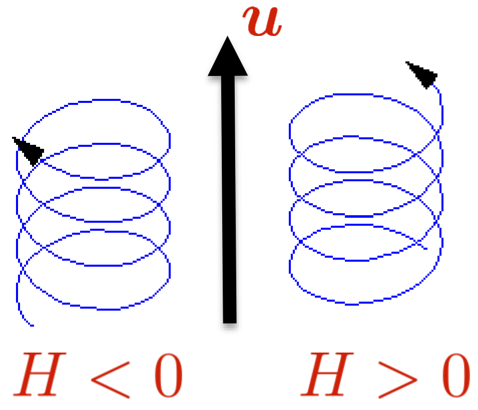
$|\Omega_z| \gg |u_z|$



Where do particles go?

Inertial particles sample the flow preferentially (e.g. spiral out if vortices)

Local helicity $H \equiv 2\boldsymbol{u} \cdot \boldsymbol{\Omega}$



Moments $\langle H^n \rangle$

↓

Distribution $P(H)$

↓

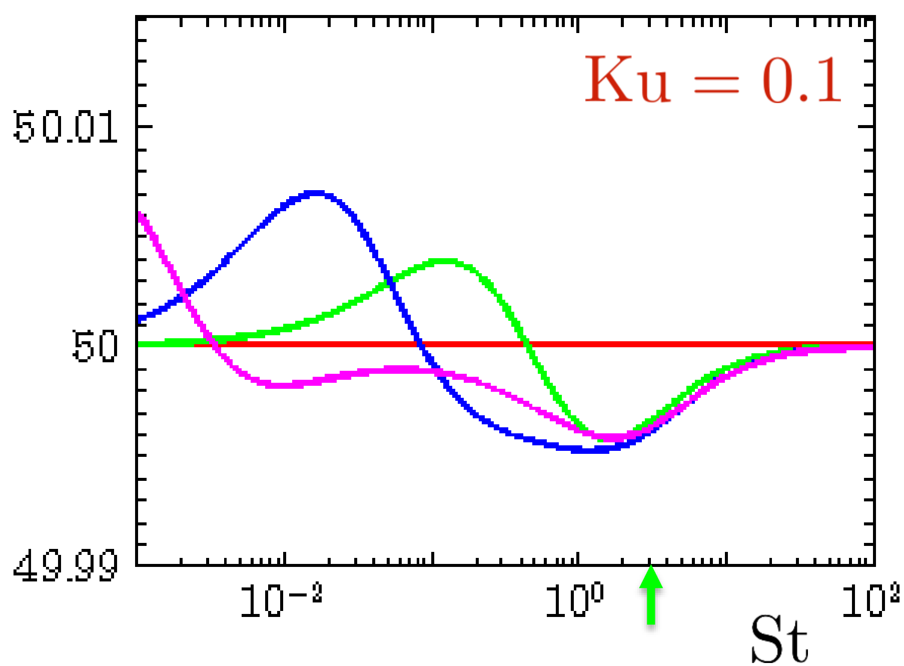
Probability to be in region with negative helicity

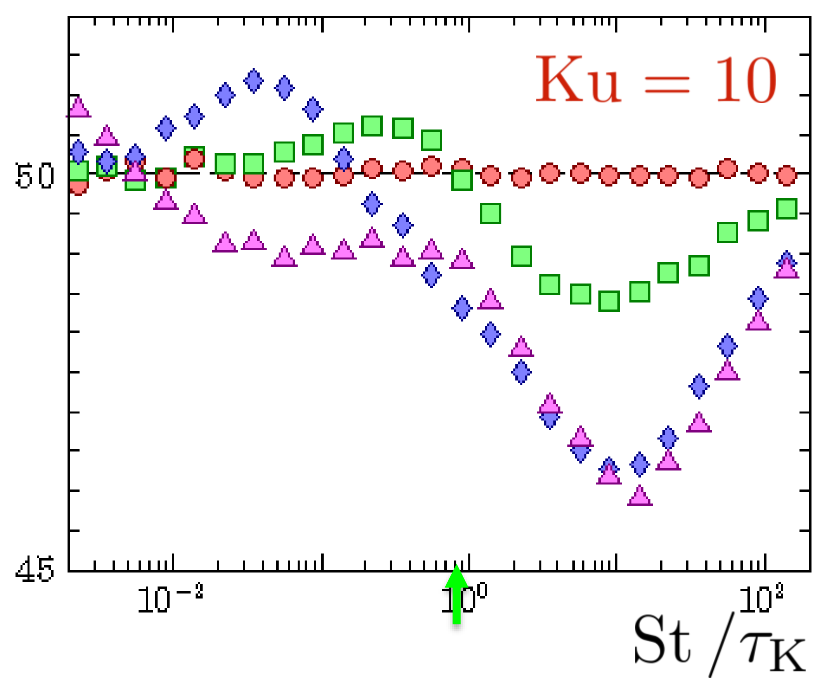
$$P(H<0) = 0.5 + \overline{a}\, C_0\, \mathrm{Ku}^2\, \mathrm{St}$$

$$\times \frac{15\sqrt{5}(50C_0^4 - 729(10+3\,\mathrm{St})(-5+\mathrm{St}(5+6\,\mathrm{St})) + 135C_0^2(-20+\mathrm{St}(7+15\,\mathrm{St})))}{2\pi(5C_0^2 - 27(1+2\,\mathrm{St})(5+3\,\mathrm{St}))(10C_0^2 - 27(1+\mathrm{St})(10+3\,\mathrm{St}))^2}$$

Probability of negative helicity

$P(H > 0)\ \ (\%)$ $\quad C_0 = 0 \quad C_0 = 3 \quad C_0 = 5 \quad C_0 = 5.19$





Small-Ku theory — Numerical data

$$P(H<0) = 0.5 + \overline{a}\, C_0\, \mathrm{Ku}^2\, \mathrm{St} \times \frac{15\sqrt{5}(50C_0^4 - 729(10+3\,\mathrm{St})(-5+\mathrm{St}(5+6\,\mathrm{St})) + 135C_0^2(-20+\mathrm{St}(7+15\,\mathrm{St})))}{2\pi(5C_0^2 - 27(1+2\,\mathrm{St})(5+3\,\mathrm{St}))(10C_0^2 - 27(1+\mathrm{St})(10+3\,\mathrm{St}))^2}$$

Conclusions

Isotropic helicoids are rotation invariant particles which break reflection invariance (two chiralities)

Coupling between translational and rotational degrees of freedom changes dynamics compared to spherical particles (modified clustering, preferential sampling etc.)

The two chiralities may show different dynamics if the particle size is not too small and flow is persistent

Flows with broken parity invariance increase the differences in the dynamics of the two particles